JBUON 2017; 22(5): 1307-1313
ISSN: 1107-0625, online ISSN: 2241-6293 • www.jbuon.com
E-mail: editorial_office@jbuon.com

ORIGINAL ARTICLE

The impact of dose matrix and voxel dimensions to dose calculation for stereotactic radiosurgery

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Summary

Purpose: *The purpose of the present study was the optimization of dose calculation for intracranial stereotactic radiosurgery.*

Methods: *We evaluated the Singular Value Decomposition (SVD) analysis as a novel optimization technique. Our approach to dose distribution optimization was to recover estimates of minibeams weights from well-defined provisional dose matrices. The dose delivered by a set of minibeams is formulated as a matrix equation. We studied the influence of dose matrix and voxel dimensions on the conditioning of stereotactic radiotherapy inverse problem. Dose matrix dimensions varied from 16 to 96 mm^3, while voxel dimension was kept constant at 2 mm^3. In the assessment of voxel dimension, matrix dimension was kept constant at 80 mm^3 while voxel dimensions varied from 1 to 8 mm^3. The reconstruction of dose distributions was studied using a truncated SVD expansion in the calculation of approximation to the generalised matrix inverse.*

Results: *The conditioning was deteriorated by either the decreasing of dose matrix dimensions or by the increasing of voxel size. The condition number was equal to 89×10^3 and 7×10^3 for the 16 mm^3 and 96 mm^3 dose matrix dimensions, respectively. The condition number was equal to 9.9×10^3 and to 2.7×10^6 for 1 mm^3 and 8 mm^3 voxel size, respectively. The reconstruction of dose distributions revealed that an ill-conditioned problem yields poor quality reconstruction.*

Conclusion: *We considered that a good compromise between quality of dose distribution, time calculation and hard disk memory would be the use of a 64 mm^3 matrix dimension with a 2 mm^3 voxel size.*

Key words: *optimization, radiation physics, stereotactic radiosurgery, three dimensional dose matrix*

Introduction

Brain tumours and arteriovenous malformations with simple elliptical or more complex shapes treated by stereotactic irradiation using converging beams of small dimensions (minibeams) could be regarded as candidates for precision conformal radiotherapy [1,2]. In stereotactic radiosurgery, the development of new optimisation techniques is as crucial as in conventional radiotherapy. We therefore propose a novel optimisation technique, namely the SVD [3-7]. The SVD analysis takes into account all the voxels of the 3-dimensional (3D) dose matrix for the calculation of the dose distribution and allows the measurement of the ill-conditioning of the stereotactic radiosurgery problem.

Radiation therapy treatment planning using 3D patient data can be a time-consuming process because of the 3D dose calculation in a volume of interest with a uniformly distributed matrix of points. Some studies have been carried out comparing the efficiency of two commonly used sampling methods in the evaluation of treatment

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Received: 13/03/2017; Accepted: 29/03/2017

plans: the regular grid point method (Cartesian method) and the pseudo-random number method [8,9].

The aim of this study was to examine how dose matrix and voxel dimensions (for regular Cartesian grids) influence the inverse treatment planning optimization process and the obtained dose distributions for stereotactic radiotherapy.

Methods

The irradiation and the dose space

The irradiation space (Figure 1) is defined by two angular apertures: Anterior-Posterior=180° and Right-Left =180°.

The Anterior-Posterior aperture is sampled in arcs with an angular sampling of $\Delta\varphi$=10°. Each arc is sampled in elementary beams called minibeams with an angular sampling of $\Delta\theta$=10°. Thus the irradiation is constituted of 361 minibeams.

The dose delivered by a set of minibeams is calculated in a dose matrix D (Figure 1). In study A dose matrix dimensions varied from 16 to 96 mm, voxel dimension was constant to 2 mm (Table 1). In the study of voxel dimension (study B), matrix dimension was constant to 80 mm while voxel dimensions varied from 1 to 8 mm (Table 1).

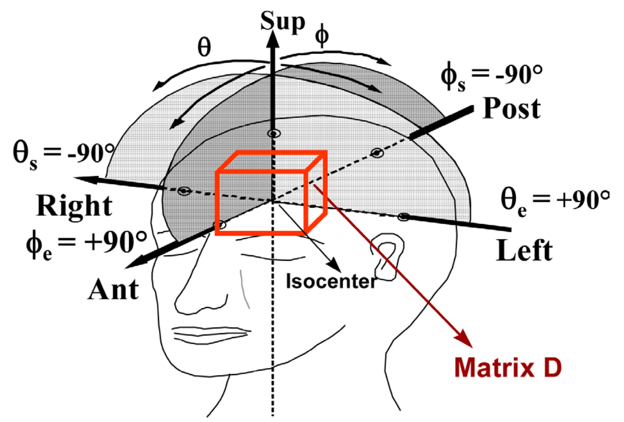


Figure 1. The irradiation space for a sitting patient position.

Table 1. Values of dose matrix and voxel dimensions studied

Study	*A*	*B*
Voxel size (mm)	2 (constant)	1, 2, 4, 8
Matrix dimension (mm)	16, 32, 48, 64, 80, 96	80 (constant)

Modelling of convergent multi-arcs stereotactic irradiation

If the irradiation space is focused on to a volume, then the dose D delivered by NM converging minibeams at a same isocenter is described by the linear system of equations.

$$D_\lambda = \sum_{i=1}^{NM} M_{\lambda,i} \times W_i \qquad \lambda = 1,2, \ldots NV \quad i = 1,2, \ldots NM \qquad [1]$$

λ: the index of the 3-D dose matrix voxel, D_λ: the calculated dose at this voxel, i: the index of the beams, W_i: the weight of beams, $M_{\lambda,i}$: the kernel matrix M

$$D = M \times W \rightarrow W = M^{-1} \times D \qquad [2]$$

Singular Value Decomposition and generalized matrix inversion

Any real NVxNM matrix M (NV>NM) may be expressed in the form:

$$M = USV^T \qquad [3]$$

$U^TU=V^TV=In$, and $S=\text{diag}(\sigma_1,\ldots\sigma_p)$, $\sigma_p>0$, p=min(NV, NF), σ_p: singular value, T: denotes matrix transpose.

The condition number R of M

Inverse problems are usually ill-conditioned. The condition number R measures the sensitivity of a solution of a linear system to different variations and gives an indication of the stability of the inverse problem (when the problem is ill-conditioned, R is large).

$$\left(R = \frac{\text{largest singular value}}{\text{smallest singular value}}\right) \qquad [4]$$

Regularization of the ill-posed problems

The importance is to find the best rank L ($1 \le L \le NM$) matrix approximation to the matrix M which removes the noise :

$$W=(M^TM)^{-SVD} \times M^T \times D = \left(\sum_{i=1}^{L} \frac{1}{\sigma_n{}^2} v_n v_n{}^T\right) M^T D \qquad [5]$$

The methodology of a predefined weighting vector reconstruction

Our approach to dose distribution optimisation is to recover estimates of beam weights from a well predefined dose matrix (PDM).

The procedure consists of:

i. Choosing arbitrary anterior-posterior (AP) and right-left (RL) angular apertures to create a Predefined Irradiation Space (PIS) (ex. PIS: $(AA)_{AP} \times (AA)_{RL}$= [-60°+60°] x [-50°+50°], Figure 2a);

ii. Determining the weights for the beams which constitute the PIS. This beam weighting vector will be known as a Predefined Weighting Vector (PWV) (W=1 for the beams $\in$ ([-60°+60°] x [-50°+50°]) and W=0 for the beams $\notin$ ([-60°+60°] x [-50° +50°]) Figure 2b);

iii. Using the PWV we compute the corresponding PDM (Figure 2c) having as tool the ARTEMIS-3D TPS (the computation of PDM is a forward calculation);

iv. Running the "SVD optimizer" software with the PDM as input data to find the Reconstructed Weighting Vector (RWV) closer to PWV (inverse calculation). The number of RWVs is equal to the number of the discredited minibeams. For each RWV the reconstruction error δ is computed:

$$\delta_L = \sqrt{\frac{\sum_{\lambda=1}^{NV} (PWV_\lambda - RWV_\lambda(L))^2}{NM}} \quad L= 1,2, ...NM \qquad [6]$$

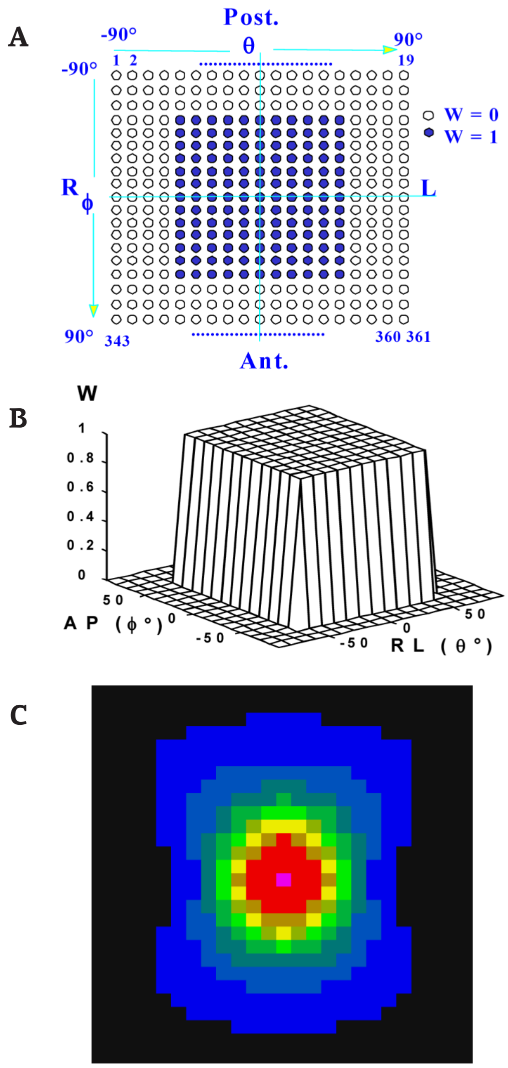


Figure 2. The definition of **(A)** PIS, **(B)** PWVand **(C)** PDM (frontal plane).

Results

Singular Value Decomposition

The superposition matrix M^TM

The shape of the superposition minibeams matrix M^TM reflects the physical properties of each configuration. It is the shape and the complexity of each M^TM that influence the ill-conditioning of the space. Figure 3 presents three M^TM as a function of the dose matrix dimensions. One observes that in the case of a 16 mm dose matrix (Figure 3a) the M^TM is rather homogeneous as the dose is computed in a region very close the isocenter and thus the voxels deliver almost the same value. The greater the dose matrix dimensions are, the more the inhomogeneous the superposition matrix becomes (Figure 3b, c), as the dose computation is realized in a larger 3D region, which means that even the few interactions between the minibeams are being considered in the computation process.

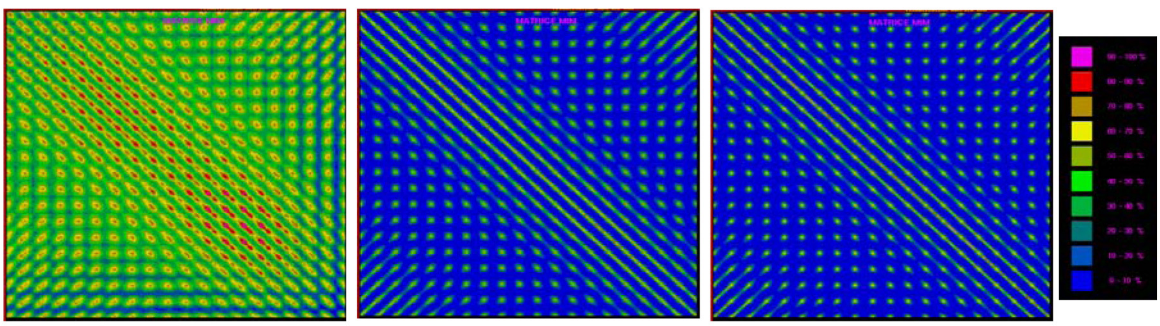
In the study of voxel size the allure of the superposition matrix M^TM was constant as the dose was computed always for the same region (dose matrix of 80 mm), as shown in Figure 4.

However, in the case of a small voxel size the superposition matrix is more detailed due to the larger number of coded information.

The singular values spectra and their condition numbers

The singular value spectrum of the matrix M for each configuration is presented in Figure 5. The shape of the spectrum defines the condition number R (Tables 2 and 3).

The results showed that the conditioning is deteriorated by the decreasing of dose matrix dimensions and the increasing of voxel size.



16 mm A | 48 mm B | 96 mm C

Figure 3. The minibeams superposition matrix for **(A)** 16 mm, **(B)** 48 mm and **(C)** 96 mm dose matrix dimension.

Reconstruction

In order to examine the influence of dose matrix dimension and voxel size on inverse treatment plan we studied the reconstruction of the PWV presented in Figure 2b.

For each configuration we computed and examined the reconstruction error curve δ as a function of the number of singular components. The study of the curves δ permits us to obtain the optimum value of the reconstruction error δ^* for each configuration. Figure 6 presents the δ^* for the different examined cases.

The reconstruction of dose distributions revealed that an ill-conditioned problem (low value of dose matrix dimension and big voxel size) yields poor quality reconstruction. Figures 7 and 8 present the optimal reconstructed weighting vector (PWV) obtained by an inverse treatment planning in some configurations.

Discussion and conclusion

Stereotactic radiosurgery is now a well known technique used for selectively destroying small intracranial lesions. Optimization procedures have also been developed to improve dose distribution. Unfortunately the optimization process is usually very time-consuming because of the 3D dose calculation. Several studies investigated and devel-

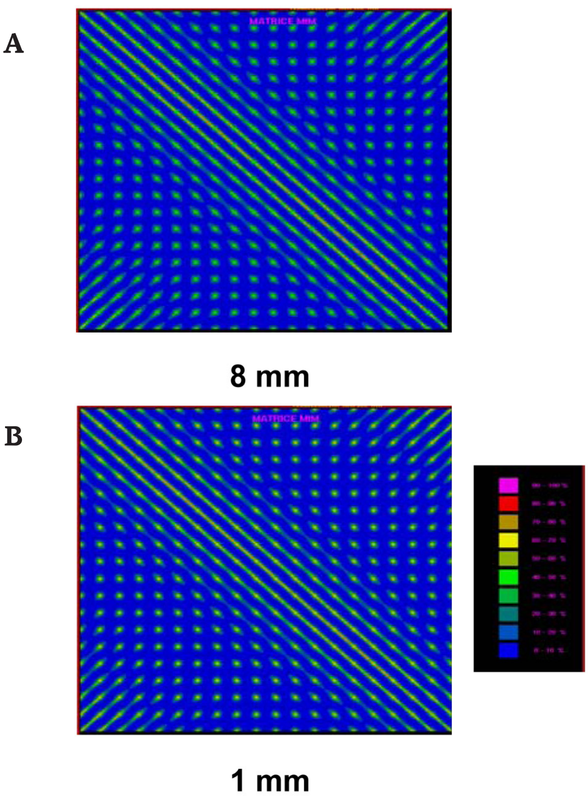


Figure 4. The minibeams superposition matrix for **(A)** 8 mm and **(B)** 1 mm voxel size.

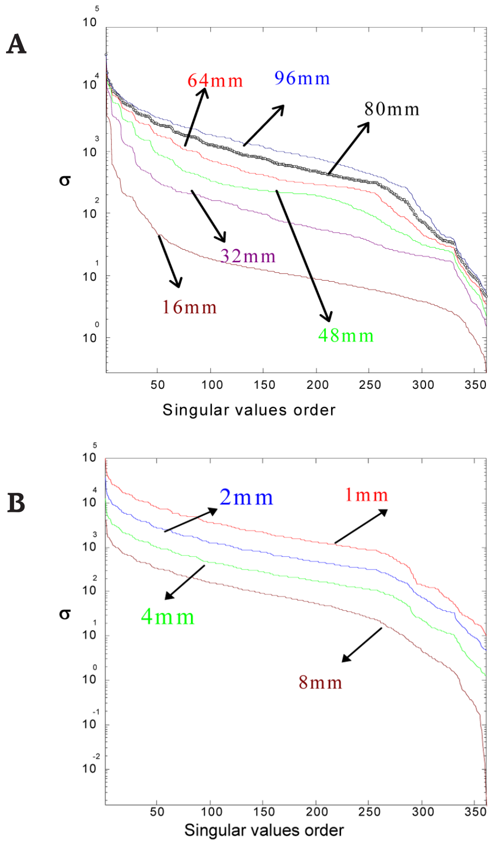


Figure 5. The singular values spectra as a function of the **(A)** dose matrix dimension and **(B)** voxel size.

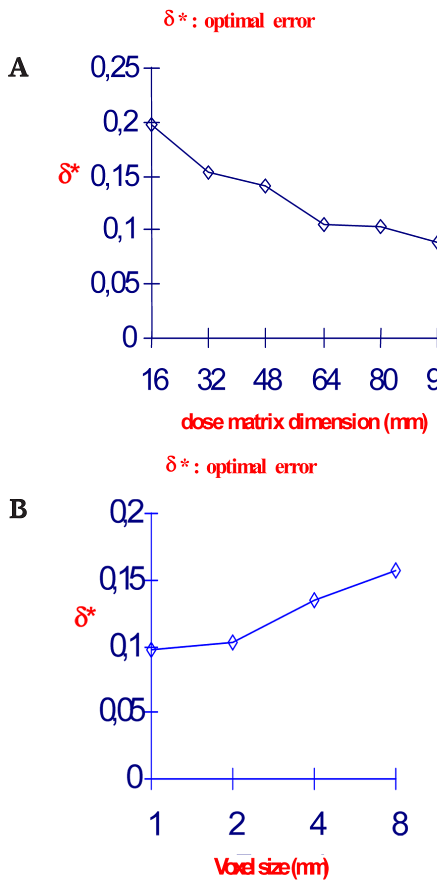


Figure 6. The optimum reconstruction error δ* as a function of the **(A)** dose matrix dimension and **(B)** voxel size.

Table 2. The condition number R as a function of the dose matrix dimension

Dose matrix dimension (mm)	*16*	*32*	*48*	*64*	*80*	*96*
Max. singular value	24,466	31,546	33,790	35,130	36,110	36,879
Min. singular value	0.27	1.53	2.3	3.43	4.71	5.25
Condition number R	89,622	20,589	15,127	10,241	7,657	7,011

Table 3. The condition number R as a function of the voxel size

Voxel size (mm)	*1*	*2*	*4*	*8*
Max. singular value	102,071	36,110	12,785	4,293
Min. singular value	10.28	4.71	1.24	0.0015
Condition number R	9,928	7,657	10,307	2,768,589

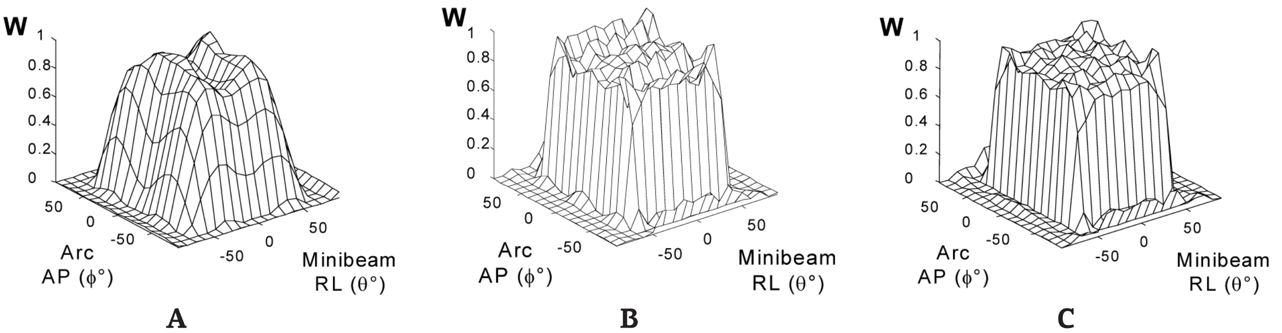


Figure 7. The RWV in the cse of **(A)** 16 mm, **(B)** 48 mm and **(C)** 96 mm dose matrix dimension.

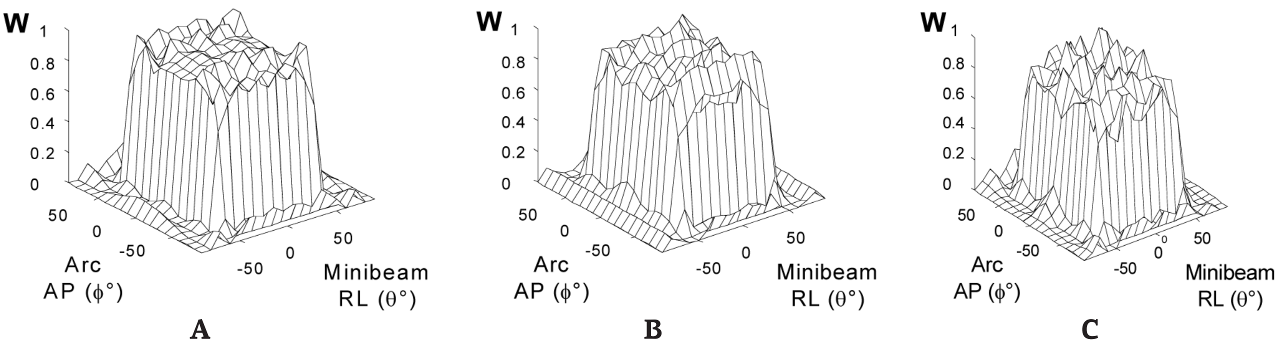


Figure 8. The RWV in the case of **(A)** 1 mm, **(B)** 4 mm and **(C)** 8 mm voxel size.

oped methods for fast dose calculation algorithms. Zhu [9] investigated three fast dose calculation algorithms for stereotactic radiosurgery treatment planning optimization. These methods included: (a) random sampling; (b) beam size adaptive ring region sampling; (c) tumor boundary oriented sampling. All three algorithms have been investigated so as to reduce the number of dose calculation points while trying to keep the calculated objective function error-free. Random sampling is a process that randomly samples a number of points from the 3D dose calculation matrix. However, random sampling does not significantly improve the speed of the dose calculation because of the associated error in the objective function calculation. As less points are sampled to speed up the dose and objective function calculation process, the error increases. The beam size adaptive ring region sampling method makes use of the following fact of the stereotactic radiosurgery dose distribution : that the size of the prescribed isodose volume changes with the sizes of the beams used. To obtain the isodose volume for the objective function calculation, a ring region which is adaptive to the beam sizes is used to replace the entire 3D dose calculation matrix. However, the speed improvement decreases for large beam sizes. The tumor boundary oriented sampling method defines a dose calculation region around the tumor boundary with some width. Based on the fact that the optimal isodose volume should conform to the target volume, defining a tumor and normal tissue ring around the tumor boundary for objective function calculation should provide the same optimal objective value. Tumor boundary oriented sampling significantly improves the speed of the objective function.

A random sampling method has also been proposed by Lam [10] to improve the speed of the dose calculation. Unfortunately, the sampled values of an objective function are different from one sample to another. Such a sampling method cannot be used in automatic optimization because the next move in an optimization process is based on the current and past objective function values. To this end, an adaptive method based on the size of the collimators is proposed and used to determine a small volume in the shape of a hollow sphere for which the dose is calculated. With an appropriate choice of an adaptive hollow sphere, the objective function calculated based on such a hollow sphere is the same as that calculated with the traditional 3D cube matrix. However, with the new adaptive method, the speed of calculating a dose can be improved by a factor of 4 to a factor of 100.

Recent technological advances in the field have shifted the scientific interest and have utilised the use of LINAC-based volumetric modulated arc therapy –VMAT- as an additional way to treat single and/or mulriple intracranial lesions. Therefore much work is currently underway in the field of automated linac-based treatment planning optimization yielding promising results [11-15]. Nonetheless complicated methods should be tested dosimetricaly [16] and undergo specific quality assurance tests [17] before they can be applied safely to clinical routine.

According to our results, the use of large dose matrix dimensions and a small voxel size do not improve the inverse optimization of dose distribution in stereotactic radiotherapy. A good compromise between quality of dose distribution, time calculation and hard disk memory is the use of a 64 mm^3 matrix dimension with a 2 mm^3 voxel size. The validation of these results has to be made by the study of clinical cases. With the newly developed fast dose calculation methods, treatment dose planning optimization for stereotactic radiosurgery should become a routine process in a clinical setting because of the speed improvement in dose calculation. Because of the improvement in the speed of calculating a treatment dose, the new adaptive hollow sphere method for calculating a treatment dose can be used routinely in designing a treatment plan.

Conflict of interests

The authors declare no confict of interests.

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